# Distributed Control for Vehicle Platoon Systems Subject to Heterogeneous Time-Varying Delays

Arezou Elahi

***Abstract:*** **This study investigates a delay-independent consensus control strategy for vehicular platooning systems subject to time-varying network delays. First, we design a sliding manifold that ensures the convergence of distance tracking errors to zero within a prescribed time interval once sliding occurs. Next, by applying Lyapunov theory, we formulate a distributed control law that steers the system states onto the manifold within a fixed settling time. The theoretical developments are ultimately validated through simulation experiments.**

***Keywords:*** **Distributed control, vehicular platooning system, time delay, uncertainty.**

## I Introduction

The primary goal of platoon control systems is to ensure that vehicles travel at a common speed while preserving a safe inter-vehicle distance [1]. Such a configuration not only optimizes traffic flow and enhances road safety but also helps reduce overall exhaust emissions [2]. Recent studies have explored various facets of vehicular platooning [3]-[5]. Within this landscape, communication topology emerges as a pivotal factor in platoon control design [7]. In contrast to adaptive cruise control, cooperative vehicular control systems necessitate both onboard sensing and vehicle-to-vehicle communication to ensure comprehensive safety across the system, thereby enabling superior performance and robust platoon coordination.

Communication topologies for vehicular networks are typically categorized as either directed or undirected [8]. The main feature of our proposed scheme is its topology nature—it operates effectively regardless of whether the graph is directed or undirected, and this versatility serves as the primary impetus for this study. Nevertheless, practical deployments introduce substantial difficulties, including dynamic topology switching, time-varying transmission lags, nonlinear vehicle behavior, and stochastic packet losses. In this context, vehicular platoons can be modeled as multi-agent systems, for which network-based distributed control strategies offer a viable solution. This perspective has given rise to several advanced approaches, such as those in [9], [10].

Sliding mode control (SMC) is extensively utilized in controller synthesis [11]-[13] due to its proven efficacy against nonlinearities, uncertainties, topological variations, and disturbances, ensuring precise tracking and fast convergence [14]. Relevant prior works include distributed SMC designs for platoons with various general topologies, to name just a few [15]-[17]. The concepts of finite-time and fixed-time stability have been extensively studied for numerous classes of dynamical systems. In these paradigms, the settling time serves as a key performance indicator, measuring how swiftly the controlled response converges [19], [20]. In practice, faster convergence is generally advantageous for improved dynamic behavior. A fundamental drawback of finite-time stabilization, however, lies in its dependence on the initial states when estimating the convergence bound [21]-[23]. Since these initial conditions are seldom available in multi-agent platforms, this dependency severely limits practical applicability. Fixed-time control overcomes this limitation by decoupling the settling time from the initial conditions [24], [25]. Regrettably, however, nearly all published works in this area assume perfect network conditions, effectively disregarding transmission delays and other communication imperfections [26], [27]. The present study bridges this gap by formulating a fixed-time controller for vehicle platoons subject to time-varying communication delays.

Motivated by the aforementioned discussions, this work develops a consensus control scheme for heterogeneous platoons under general topologies by designing an observer-based predefined-time sliding-mode controller.

### A Organization

The remainder of this paper is organized as follows. Section II and III introduce the preliminaries and the problem formulation, respectively. Section IV presents the proposed observer, the controller, and the associated stability analysis. Numerical simulations are provided in Section V, followed by concluding remarks in Section VI.

### B Notation

We employ the following mathematical notations: $\mathbb{R}^n$ and $\mathbb{R}^{N\times N}$ denote, respectively, the sets of real $n$-component vectors and real $N\times N$ matrices, $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of a given matrix, $I_n$ indicates the identity matrix of order $n$, and for $\sigma \geq 0$, $x \in \mathbb{R}^n$, the function $\lceil y \rfloor^{\sigma} = |x|^{\sigma}\text{sign}(y)$ is used. Finally, $\|X\|$ is adopted throughout to represent the Euclidean norm of a vector or matrix $X$.

Manuscript received 1 July 2026; revised 24 July 2026; accepted 3 August 2026.

A. Elahi is with the faculty of electrical and computer engineering, University of Gonabad, Gonabad, Iran (e-mail: arezou_elahi@yahoo.com).

## II Preliminaries

First, we state the preliminaries that will be used in the manuscript.

### A Graph theory

Suppose that the symbol $G = (\mathcal{V}, \mathcal{E})$ shows the communication topology among $N$ followers, where $\mathcal{V} = \{1, 2, ..., N\}$ denotes the set of followers and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represents the set of edges. $N_i$ indicates the set of neighbors of follower $i$. If there is a link from follower $j$ to follower $i$, then $(j, i) \in \mathcal{E}$. In addition, $A = [a_{ij}]$ denotes the adjacency matrix in which if $(i, j) \in \mathcal{E}$, then $a_{i,j} > 0$, otherwise $a_{ij} = 0$. The Laplacian matrix is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where the elements satisfy $l_{ii} = \sum_{j\in N} a_{ij}$, and $l_{ij} = -a_{ij}$, where $i \neq j$. Define $\mathcal{M} = diag\,(g_i)$ as the diagonal pinning matrix, where $g_i = 1$ if we have a directed edge from the leader to the follower, and $g_i = 0$ otherwise. Notice that there is at least one edge connecting leader and followers.

**Lemma 1 [28]:** Let us consider the system with dynamical equation

$$\dot{\xi}(t) = -\lambda_1 \lceil \xi(t) \rceil^{\rho_1} - \lambda_2 \lceil \xi(t) \rceil^{\rho_2} \tag{1}$$

with $\lceil \xi(t) \rceil^{\rho_1} = \text{sign}\,(\xi(t))|\xi(t)|^{\rho_1}$, $\lambda_1$, $\lambda_2 > 0$, $1 > \rho_2 > 0$, $\rho_1 > 1$. Thus, for the system (1), fixed-stability is guaranteed with the maximum admissible value of setting time $T = \frac{1}{\lambda_1(\rho_1-1)} + \frac{1}{\lambda_2(1-\rho_2)}$.

**Lemma 2 [29]:** Let us consider the following integrator system with the initial conditions $\xi(0) = \xi_0$

$$\begin{aligned} \dot{\xi}_1(t) &= \xi_2(t) \\ \dot{\xi}_2(t) &= \xi_3(t) \\ \dot{\xi}_3(t) &= u(t) \end{aligned} \tag{2}$$

where $\xi_i(t) \in \mathbb{R}^n$, $i = 1, 2, 3$, and $u(t)$ represent the system's states and the control signal, respectively. Assume the polynomial $s^3 + k_3 s^2 + k_2 s + k_1$ is Hurwitz, in which $s$ denotes the Laplace operator and $k_i > 0$, $i = 1, 2, 3$ are the positive constants. There exists a scalar $\breve{\gamma} \in (\frac{1}{2}, 1)$ so that, for $\vartheta \in (\breve{\gamma}, 1)$, the system (2) in a fixed-time is stabilized at the origin via the following control law

$$u(t) = -\sum_{i=1}^{3} k_i \left( \lceil \xi_i(t) \rceil^{\alpha_i} + \lceil \xi_i(t) \rceil + \lceil \xi_i(t) \rceil^{\tilde{\alpha}_i} \right) \tag{3}$$

in which the parameters $\alpha_i$ and $\tilde{\alpha}_i$ satisfying $\alpha_{3-j} = \frac{\vartheta}{(j+1)-j\vartheta}$, $\tilde{\alpha}_{3-j} = \frac{2-\vartheta}{j\vartheta-(j-1)}$, $j = 0, 1, 2$.

**Lemma 3 [29]:** For the positive parameters $\eta_i$, $i = 1, ..., N$, $1 \geq \alpha_1 > 0$, $\alpha_2 > 1$, the following relationships hold

$$\begin{aligned} N^{1-\alpha_2} \left( \sum_{i=1}^{N} \eta_i \right)^{\alpha_2} &\leq \sum_{i=1}^{N} \eta_i^{\alpha_2} \\ \left( \sum_{i=1}^{N} \eta_i \right)^{\alpha_1} &\leq \sum_{i=1}^{N} \eta_i^{\alpha_1} \end{aligned} \tag{4}$$

## III Problem statement

Similar to most of the existing literature, we adopt the heterogeneous third-order model for $i$-th vehicle as [6]

$$\begin{aligned} \dot{q}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= a_i(t) \\ \dot{a}_i(t) &= -\frac{1}{\tau_i} a_i(t) + \frac{1}{\tau_i} u_i(t) + f_i(t, q_i, v_i, a_i) \end{aligned} \tag{5}$$

where $i \in \{0, 1, \ldots, N\}$, $a_i$, $v_i$, $q_i \in \mathbb{R}^n$ represent the acceleration, velocity, and position, respectively, $u_i(t)$, $\tau_i$ indicate the control input and the time constant, respectively, and the uniformly bounded function $f_i$ satisfying $|f_i| \leq \gamma$ denotes the uncertainty term, where $\gamma$ is a certain positive constant. The rest of this article, $f_i(t)$ will adopted instead of $f_i(t, q_i, v_i, a_i)$.

Let us consider the spacing error

$$e_{q,i}(t) = \Big( -q_i(t) + q_{i-1}(t - r_i(t)) - d_i \Big) - \Big( h_1 + h_2 v_i(t) \Big) \tag{6}$$

in which $h_1$, $h_2$, and $d_i$ describe the standstill distance, desired time gap, and vehicle length, respectively. The transmission delays $r_i\,(t)$ between the vehicles $i-1$ and $i$ is time-varying and unknown bounded.

The central goal of the fixed-time consensus control problem is the derivation of control policies that enforce the consensus error states to vanish within a uniformly bounded time, independent of the system's initialization. Upon differentiating the spacing errors presented in (6), one can derive the error dynamics outlined below.

$$\begin{aligned} \dot{e}_{q,i}(t) &= \varsigma_{i,1}(t) + v_{i-1}(t - r_i(t)) + v_{i-1}(t) \\ &\quad - v_i(t) - h_2\, a_i(t) \\ \dot{e}_{v,i}(t) &= \varsigma_{i,2}(t) + a_{i-1}(t - r_i(t)) + a_{i-1}(t) \\ &\quad - a_i(t) - h_2\, \dot{a}_i(t) \\ \dot{e}_{a,i}(t) &= \varsigma_{i,3}(t) + \dot{a}_{i-1}(t) - \dot{a}_i(t) - h_2\, \ddot{a}_i(t) \end{aligned} \tag{7}$$

where

$$\begin{aligned} e_{v,i}(t) &= v_{i-1}(t - r_i(t)) + v_{i-1}(t) - v_i(t) - h_2\, a_i(t) \\ e_{a,i}(t) &= a_{i-1}(t - r_i(t)) + a_{i-1}(t) - a_i(t) - h_2\, \dot{a}_i(t) \\ \varsigma_{i,1}(t) &= -\dot{r}_i(t) v_{i-1}(t - r_i(t)) - v_{i-1}(t) \\ \varsigma_{i,2}(t) &= -\dot{r}_i(t) a_{i-1}(t - r_i(t)) \\ \varsigma_{i,3}(t) &= -\dot{r}_i(t)\, \dot{a}_{i-1}(t - r_i(t)) + \dot{a}_{i-1}(t - r_i(t)) \end{aligned} \tag{8}$$

Through straightforward simplifications, the following expression is obtained

$$\begin{aligned} \dot{e}_{q,i}(t) &= e_{v,i}(t) + \varsigma_{i,1}(t) \\ \dot{e}_{v,i}(t) &= e_{a,i}(t) + \varsigma_{i,2}(t) \\ \dot{e}_{a,i}(t) &= \Omega_i(t) + \breve{u}_i(t) + \hat{\varsigma}_{i,3}(t) \end{aligned} \tag{9}$$

where

$$\begin{aligned} \hat{\varsigma}_{i,3}(t) &= \varsigma_{i,3}(t) + f_{i-1}(t) - f_i(t) + \frac{h_2}{\tau_i} f_i(t) - h_2 \dot{f}_i(t) \\ \breve{u}_i(t) &= \Big( \frac{1}{\tau_{i-1}} u_{i-1}(t) - \frac{1}{\tau_i} u_i(t) + \frac{h_2}{\tau_i^2} u_i(t) - \frac{h_2}{\tau_i} \dot{u}_i(t) \Big) \end{aligned}$$

$$\Omega_i(t) = \Big( -\frac{1}{\tau_{i-1}} a_{i-1}(t) + \frac{1}{\tau_i} a_i(t) - \frac{h_2}{\tau_i^2} a_i(t) \Big) \tag{10}$$

## IV Stability Analysis

Note from (7)–(10) that $\varsigma_{i,1}$, $\varsigma_{i,2}$, $\hat{\varsigma}_{i,3}$ denote complex unknowns, including uncertainties and immeasurable $\dot{r}\,(t)$, that inefficient mitigation of these disturbances risks degrading the consensus capability. For this purpose, a fixed-time observer is presented in the sequel to facilitate exact observation.

**Theorem 1:** Applying the error dynamics (9), suppose that the complex unknowns $\varsigma_{i,1}(t)$, $\varsigma_{i,2}(t)$, $\hat{\varsigma}_{i,3}(t)$ are respectively bounded by $\|\dot{\varsigma}_{i,1}(t)\| < l_{i,1}$, $\|\dot{\varsigma}_{i,2}(t)\| < l_{i,2}$, $\|\dot{\hat{\varsigma}}_{i,3}(t)\| < l_{i,3}$, where $l_{i,j}$, $j = 1,2,3$ are positive constants. Under these conditions, the fixed-time observers are designed as

$$\begin{aligned}\dot{x}_{i,0}(t) = &-\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|^{\rho-1} \\ &+ x_{i,1}(t) + e_{i,v}(t), \qquad x_{i,0}(0) = 2e_{q,i}(0)\end{aligned} \tag{11}$$

$$\dot{x}_{i,1}(t) = -\Upsilon_{i,3} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|}, \qquad x_{i,1}(0) = \varsigma_{i,1}(0) \tag{12}$$

$$\begin{aligned}\dot{x}_{i,2}(t) = &-\Upsilon_{i,4} \frac{n_{i,2}(t)}{\|n_{i,2}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,5}\, n_{i,2}(t) \|n_{i,2}(t)\|^{\rho-1} \\ &+ x_{i,3}(t) + e_{i,a}(t), \qquad x_{i,2}(0) = 2e_{v,i}(0)\end{aligned} \tag{13}$$

$$\dot{x}_{i,3}(t) = -\Upsilon_{i,6} \frac{n_{i,2}(t)}{\|n_{i,2}(t)\|}, \qquad x_{i,3}(0) = \varsigma_{i,2}(0) \tag{14}$$

$$\begin{aligned}\dot{x}_{i,4}(t) = &-\Upsilon_{i,7} \frac{n_{i,4}(t)}{\|n_{i,4}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,8}\, n_{i,4}(t) \|n_{i,4}(t)\|^{\rho-1} \\ &+ x_{i,5}(t) + \Omega_i(t) + \breve{u}_i(t), \qquad x_{i,4}(0) = 2e_{a,i}(0)\end{aligned} \tag{15}$$

$$\dot{x}_{i,5}(t) = -\Upsilon_{i,9} \frac{n_{i,4}(t)}{\|n_{i,4}(t)\|}, \qquad x_{i,5}(0) = \hat{\varsigma}_{i,3}(0) \tag{16}$$

where $\rho > 1$, $x_{i,j}$, $j = 1, ..., 5$ are the state variable of observers, and the auxiliary variables are established as $n_{i,0}(t) = x_{i,0}(t) - e_{q,i}(t), n_{i,1} = x_{i,1}(t) - \varsigma_{i,1}(t)$, $n_{i,2}(t) = x_{i,2}(t) - e_{v,i}(t)$, $n_{i,3}(t) = x_{i,3}(t) - \varsigma_{i,2}(t)$, $n_{i,4}(t) = x_{i,4}(t) - e_{a,i}(t), n_{i,5}(t) = x_{i,5}(t) - \hat{\varsigma}_{i,3}(t)$, and the parameters are satisfied, where

$$\Upsilon_{i,7} > \sqrt{2\Upsilon_{i,9}},\ \Upsilon_{i,8} > 0,\ \Upsilon_{i,9} > 4\, l_{i,3} \tag{17}$$

$$\Upsilon_{i,4} > \sqrt{2\Upsilon_{i,6}},\ \Upsilon_{i,5} > 0,\ \Upsilon_{i,6} > 4 l_{i,2} \tag{18}$$

$$\Upsilon_{i,1} > \sqrt{2\Upsilon_{i,3}},\ \Upsilon_{i,2} > 0,\ \Upsilon_{i,3} > 4 l_{i,1} \tag{19}$$

Then, the mismatched disturbances $\varsigma_{i,1}(t), \varsigma_{i,2}(t)$, and unknown term $\hat{\varsigma}_{i,3}(t)$ is exactly observed within a fixed-time independent of the initial states. That is, there exists a fixed-time $T_{fj}$, $j = 1,2,3$, so that, $x_{i,1} = \varsigma_{i,1}$, $x_{i,3} = \varsigma_{i,2}$, and $x_{i,5} = \hat{\varsigma}_{i,3}$, for $t \geq T_0$, where $T_0 = \max(T_{f1}, T_{f2}, T_{f3})$.

**Proof:** We first present the criteria that ensure finite-time convergence. Building on this, we prove that the system converges within a fixed time.

Applying (6), (11), and (12), we have

$$\begin{aligned}\dot{n}_{i,0}(t) &= -\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|^{p-1} \\ &\quad + x_{i,1}(t) + e_{i,v}(t) - e_{i,v}(t) - \varsigma_{i,1}(t) \\ &= -\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|^{p-1} \\ &\quad + n_{i,1}(t), \ \ n_{i,0}(0) = e_{i,q}(0)\end{aligned} \tag{20}$$

$$\dot{n}_{i,1}(t) = -\Upsilon_{i,3} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|} - \dot{\varsigma}_{i,1}(t), \ \ n_{i,1}(0) = 0 \tag{21}$$

In the first step, we establish the following Lyapunov function for (20) and (21)

$$\begin{aligned}V(t) = &\, 2\Upsilon_{i,3} \|n_{i,0}(t)\| \\ &+ \frac{1}{2} \|n_{i,1}(t)\|^2 + \frac{1}{2} \left\| \Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - n_{i,1}(t) \right\|^2\end{aligned} \tag{22}$$

Through straightforward algebraic rearrangement, the derivative of (22) with respect to time is given by

$$\dot{V}(t) \leq -\frac{z^T \psi z}{\|n_{i,0}(t)\|^{\frac{1}{2}}} \leq -\lambda_{\min}(\psi) \frac{\|z\|^2}{\|n_{i,0}(t)\|^{\frac{1}{2}}} \tag{23}$$

where $z = col(\|n_{i,0}(t)\|^{\frac{1}{2}}, \|n_{i,1}(t)\|)$ and the matrix $\psi$ is positive definite, if

$$\Upsilon_{i,1}^2 > 2l_{i,1}, \ \ \Upsilon_{i,3} \geq 3l_{i,1} + \frac{2l_{i,1}^2}{\Upsilon_{i,1}^2} \tag{24}$$

It is worth emphasizing that condition (24) is not only sufficient for guaranteeing the convergence of the observer error dynamics, but also guarantees the positive definiteness of matrix $\psi$. Specifically, by selecting the observer gains such that $\Upsilon_{i,1}$ and $\Upsilon_{i,3}$ dominate the corresponding disturbance upper bounds, every principal minor of $\psi$ becomes strictly positive, yielding $\lambda_{min}(\psi) > 0$. Then, defining $Z = col\left( \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}}, \|n_{i,1}(t)\| \right)$, due to since $\|Z\| = \|z\|$, we have

$$\dot{V}(t) \leq -\lambda_{\min}(\psi) \frac{\|Z\|^2}{\|n_{i,0}(t)\|^2} \tag{25}$$

Considering a symmetric positive-definite matrix $\mathcal{P}$

$$\mathcal{P} = \begin{bmatrix} \left(2\Upsilon_{i,3} + \frac{\Upsilon_{i,1}^2}{2}\right) I_n & -\frac{\Upsilon_{i,1}}{2} I_n \\ -\frac{\Upsilon_{i,1}}{2} I_n & I_n \end{bmatrix} \tag{26}$$

and constructing the Lyapunov function as $V = Z^T \mathcal{P}\, Z$, we have $\sqrt{\lambda_{\min}(\mathcal{P})} \|Z\| \leq V^{\frac{1}{2}} \leq \sqrt{\lambda_{\max}(\mathcal{P})} \|Z\|$ and then

$$\dot{V} \leq -\hat{\rho} V^{\frac{1}{2}} \tag{27}$$

in which $\hat{\rho} = \frac{\lambda_{\min}(\psi)\sqrt{\lambda_{\min}(\mathcal{P})}}{\lambda_{\max}(\mathcal{P})}$. From (27), we achieve the convergence of the systems' states (20) and (21) to the origin. Notice that due to $\Upsilon_{i,1}^2 > 2l_{i,1}$, we have $\frac{1}{\Upsilon_{i,1}^2} < \frac{1}{2l_{i,1}}$

the second condition in (24) holds if $\Upsilon_{i,3} > 4l_{i,1}$.

Given that $\Upsilon_{i,3} > l_{i,1}$, the first condition is given by $\Upsilon_{i,1}^2 > 2\Upsilon_{i,3}$. Therefore, the finite-time convergence conditions (24) is expressed in the form $\Upsilon_{i,3} > 4l_{i,1}$ and $\Upsilon_{i,1}^2 > 2\Upsilon_{i,3}$.

In the second step, invoking the findings of [30], we deduce that the signals $n_{i,0}(t)$ and $n_{i,1}(t)$ in (20) and (21) are uniformly driven to zero over a fixed time horizon. That is, there exists a fixed-time $T_{f1}$ irrespective of initializations such that $n_{i,0}(t) = n_{i,1}(t) = 0$, $t \geq T_{f1}$. Because of the similarity, the proof of $n_{i,3}(t) = 0$, $t \geq T_{f2}$ and $n_{i,5}(t) = 0$, $t \geq T_{f3}$, are omitted herein. It follows that disturbances $\varsigma_{i,1}(t)$, $\varsigma_{i,2}(t)$, $\hat{\varsigma}_{i,3}(t)$ can be exactly observed with in $T_0 = \max(T_{f1}, T_{f2}, T_{f3})$. This completes the proof.

**Remark 1:** The observer gains $\Upsilon_{i,1}$ and $\Upsilon_{i,3}$ should be selected to be substantially larger than the estimated upper bounds of the disturbances. Larger gains typically accelerate observer convergence; however, excessively conservative selections may result in increased control effort. Thus, a balanced gain selection provides a practical compromise between convergence performance and implementation robustness. The constants $l_{i,1}$, $l_{i,2}$, and $l_{i,3}$ similarly denote conservative upper bounds on the lumped uncertainties affecting the observer dynamics. In practice, these bounds can be estimated using available system information, experimental identification, engineering specifications, or worst-case operating conditions. It is important to emphasize that the proposed observer does not require precise knowledge of the uncertainty magnitudes. Instead, sufficiently conservative upper bounds are adequate to ensure the theoretical convergence properties. Although overestimating these bounds may lead to larger observer gains and, consequently, heightened control effort, the fixed-time convergence of the observer remains guaranteed.

Next, a fixed-time control scheme is constructed using the fixed-time observer established above. Defining $p_{i,1} = e_{q,i}(t)$, $p_{i,2}(t) = x_{i,1} + e_{v,i}(t)$, $p_{i,3}(t) = \dot{x}_{i,1}(t) + x_{i,3} + e_{a,i}(t)$, the error dynamics (9) is rewritten as

$$\dot{p}_{i,1}(t) = p_{i,2}(t) \tag{28}$$

$$\dot{p}_{i,2}(t) = p_{i,3}(t) \tag{29}$$

$$\dot{p}_{i,3}(t) = \Omega_i + \breve{u}_i(t) + \ddot{x}_{i,1}(t) + \dot{x}_{i,3}(t) + x_{i,5}(t) \tag{30}$$

For developing fixed-time consensus control of vehicles, the following new variables are defined.

$$e_{p_{i,1}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,1}(t) - p_{j,1}(t)) + g_i p_{i,1}(t) \tag{31}$$

$$e_{p_{i,2}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,2}(t) - p_{j,2}(t)) + g_i p_{i,2}(t) \tag{32}$$

$$e_{p_{i,3}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,3}(t) - p_{j,3}(t)) + g_i p_{i,3}(t) \tag{33}$$

For the $i$-th agent, the sliding manifold is proposed as

$$\begin{aligned}\sigma_i(t) = p_{i,3}(t) &+ k_1 \int_0^t \left( \lceil e_{p_{i,1}}(\theta) \rfloor^{\alpha_1} + e_{p_{i,1}}(\theta) + \lceil e_{p_{i,1}}(\theta) \rfloor^{\tilde{\alpha}_1} \right) d\theta \\ &+ k_2 \int_0^t \left( \lceil e_{p_{i,2}}(\theta) \rfloor^{\alpha_2} + e_{p_{i,2}}(\theta) + \lceil e_{p_{i,2}}(\theta) \rfloor^{\tilde{\alpha}_2} \right) d\theta \\ &+ k_3 \int_0^t \left( \lceil e_{p_{i,3}}(\theta) \rfloor^{\alpha_3} + e_{p_{i,3}}(\theta) + \lceil e_{p_{i,3}}(\theta) \rfloor^{\tilde{\alpha}_3} \right) d\theta \end{aligned} \tag{34}$$

where the scalars $k_i$, $\alpha_i$, $\tilde{\alpha}_i$, $i = 1, 2, 3$, are opted based on Lemma 2. Differentiating (34) yields

$$\begin{aligned}\dot{\sigma}_i(t) = \dot{p}_{i,3}(t) &+ k_1 \left( \lceil e_{p_{i,1}}(t) \rfloor^{\alpha_1} + e_{p_{i,1}}(t) + \lceil e_{p_{i,1}}(t) \rfloor^{\tilde{\alpha}_1} \right) \\ &+ k_2 \left( \lceil e_{p_{i,2}}(t) \rfloor^{\alpha_2} + e_{p_{i,2}}(t) + \lceil e_{p_{i,2}}(t) \rfloor^{\tilde{\alpha}_2} \right) \\ &+ k_3 \left( \lceil e_{p_{i,3}}(t) \rfloor^{\alpha_3} + e_{p_{i,3}}(t) + \lceil e_{p_{i,3}}(t) \rfloor^{\tilde{\alpha}_3} \right) \end{aligned} \tag{35}$$

Considering the error dynamics (30) and sliding manifold (34) with $\eta > 1$, we can calculate the control signal as

$$\begin{aligned}\breve{u}_i(t) = &- \left( \Omega_i(t) + \ddot{x}_{i,1}(t) + \dot{x}_{i,3}(t) + x_{i,5}(t) \right) \\ &- k_3 \left( \lceil e_{p_{i,3}}(t) \rfloor^{\alpha_3} + e_{p_{i,3}}(t) + \lceil e_{p_{i,3}}(t) \rfloor^{\tilde{\alpha}_3} \right) \\ &- k_2 \left( \lceil e_{p_{i,2}}(t) \rfloor^{\alpha_2} + e_{p_{i,2}}(t) + \lceil e_{p_{i,2}}(t) \rfloor^{\tilde{\alpha}_2} \right) \\ &- k_1 \left( \lceil e_{p_{i,1}}(t) \rfloor^{\alpha_1} + e_{p_{i,1}}(t) + \lceil e_{p_{i,1}}(t) \rfloor^{\tilde{\alpha}_1} \right) \\ &- \left( \lceil \sigma_i(t) \rfloor^{1+\frac{1}{\eta}} + \sigma_i(t) + \lceil \sigma_i(t) \rfloor^{1-\frac{1}{\eta}} \right) \end{aligned} \tag{36}$$

**Remark 2:** Based on incomplete measurements, we introduce an observer (11)–(16) to observe the disturbance signals and unknown functions within a fixed-time. From Theorem 1, the mismatched disturbances $\varsigma_{i,1}$, $\varsigma_{i,2}$ and unknown term $\hat{\varsigma}_{i,3}$ is observed within a fixed time if (19)–(17) hold. Thus, $x_{i,1} = \varsigma_{i,1}$, $x_{i,3} = \varsigma_{i,2}$, $x_{i,5} = \hat{\varsigma}_{i,3}$ is applied in the control design scheme.

Here, independent of the graph topology, we assume that $(\mathcal{L} + \mathcal{M})$ is a positive definite matrix.

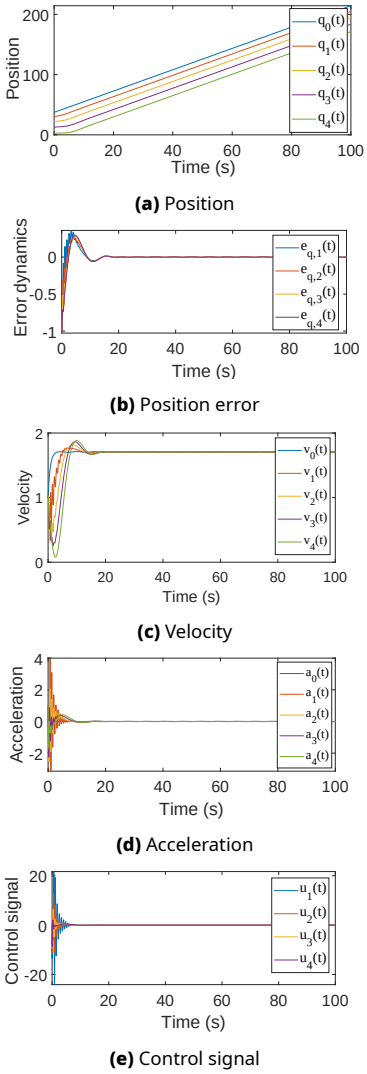
**Theorem 2:** For the error dynamics (28)-(30) under Assumption, applying the observer (11)-(16) and the control signal (36), the error dynamics converge to zero in a predefined time independent of initializations.

**Proof:** The derivative of Lyapunov function

$$V(t) = \sum_{i=1}^{N} 0.5\sigma_i^2(t) \tag{37}$$

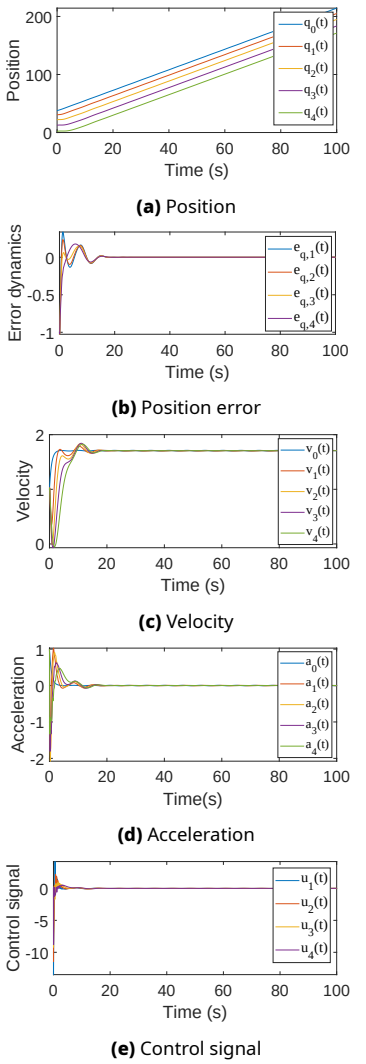
with respect to time is given by

$$\dot{V}(t) = \sum_{i=1}^{N} \sigma_i(t)\dot{\sigma}_i(t) \tag{38}$$




**(a)** Position **(b)** Position error **(c)** Velocity **(d)** Acceleration **(e)** Control signal

**Fig. 1.** Trajectories of directed topology in scenario 1.




**(a)** Position **(b)** Position error **(c)** Velocity **(d)** Acceleration **(e)** Control signal

**Fig. 2.** Trajectories of undirected topology in scenario 1.

From (30), (35), and (36), we get

$$\dot{V}(t) = -\sum_{i=1}^{N} \sigma_i(t) \left( \lceil \sigma_i(t) \rceil^{1-\frac{1}{\eta}} + \lceil \sigma_i(t) \rceil^{1+\frac{1}{\eta}} + \sigma_i(t) \right) \tag{39}$$

that satisfy

$$\dot{V}(t) \leq -\sum_{i=1}^{N} \left( |\sigma_i(t)|^{2+\frac{1}{\eta}} + |\sigma_i(t)|^{2} + |\sigma_i(t)|^{2-\frac{1}{\eta}} \right) \tag{40}$$

Applying Lemma 3, it yields

$$\dot{V}(t) \leq -( V^{1-\frac{1}{2\eta}} + N^{\frac{-1}{2\eta}} V^{1+\frac{1}{2\eta}} ) \tag{41}$$

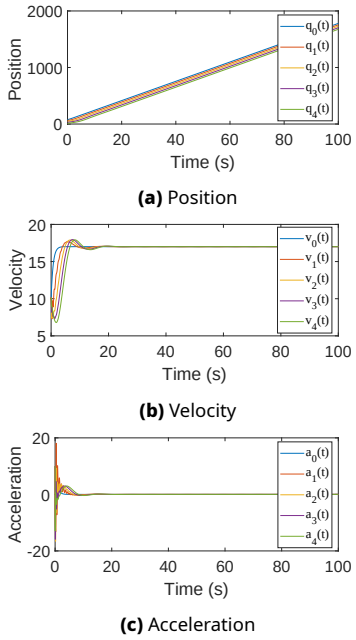
According to Lemmas 1 and 2, we ensure that every agent reaches the sliding manifold $\sigma_i(t) = 0$ within $T_r \leq 2\eta\,(1 + N^{\frac{1}{2\eta}})$ and stays there subsequently independent of initializations. In the sliding motion, $\sigma_i(t) = 0$ yields $\dot{\sigma}_i(t) = 0$. Consequently, the reduced resultant dynamics, for $t \geq T_0 + T_r$, is given by

$$\begin{aligned}
\dot{e}_{p_1}(t) &= e_{p_2}(t) \\
\dot{e}_{p_2}(t) &= e_{p_3}(t) \\
\dot{e}_{p_3}(t) &= -\tilde{\mathcal{L}} K_1 \left( \lceil e_{p_1}(t) \rceil^{\alpha_1} + e_{p_1}(t) + \lceil e_{p_1}(t) \rceil^{\tilde{\alpha}_1} \right) \\
&\quad - \tilde{\mathcal{L}} K_2 \left( \lceil e_{p_2}(t) \rceil^{\alpha_2} + e_{p_2}(t) + \lceil e_{p_2}(t) \rceil^{\tilde{\alpha}_2} \right) \\
&\quad - \tilde{\mathcal{L}}\, K_3 \left( \lceil e_{p_3}(t) \rceil^{\alpha_3} + e_{p_3}(t) + \lceil e_{p_3}(t) \rceil^{\tilde{\alpha}_3} \right)
\end{aligned} \tag{42}$$

where $\tilde{\mathcal{L}} = (\mathcal{L} + \mathcal{M})$, and for $j = 1, 2, 3$,

$$e_{p_j} = \begin{bmatrix} e_{p_{1,j}} & e_{p_{2,j}} & \vdots & e_{p_{N,j}} \end{bmatrix}^T$$

$$\lceil e_{p_j}(t) \rceil^{\sigma} = \begin{bmatrix} \lceil e_{p_{1,j}}(t) \rceil^{\sigma} \\ \lceil e_{p_{2,j}}(t) \rceil^{\sigma} \\ \vdots \\ \lceil e_{p_{N,j}}(t) \rceil^{\sigma} \end{bmatrix}, \sigma = \alpha_j, \tilde{\alpha}_j$$

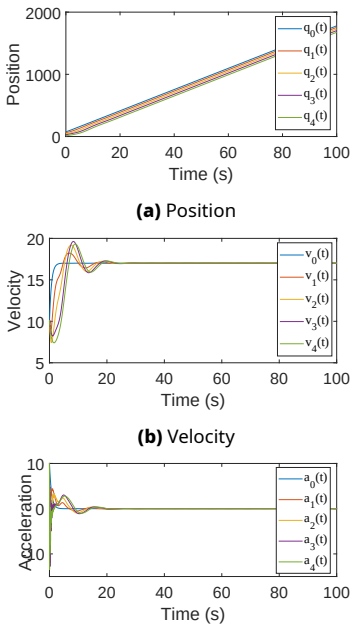



**(a)** Position

**(b)** Velocity

**(c)** Acceleration

**Fig. 3.** Trajectories of directed topology in scenario 2.




**(a)** Position

**(b)** Velocity

**(c)** Acceleration

**Fig. 4.** Trajectories of undirected topology in scenario 2.

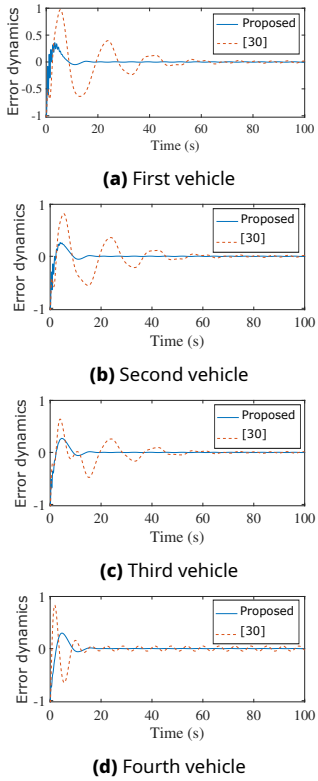
$$K_j = \begin{bmatrix} k_j & 0 & \cdots & 0 \\ 0 & k_j & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & k_j \end{bmatrix} \in \mathbb{R}^{N\times N} \succ 0 \qquad (43)$$

From Lemma 2, (42) are fixed-time stable at the origin. In other words, irrespective of the initializations and for $t \geq T_r + T_0$, we have $e_{p_j} \to 0$, $j = 1, 2, 3$, for all agents, where $T_0$ is given by in Theorem 1. Since $e_{p_j} \to 0$ indicates $p_{i,j} \to 0$, $i \in \mathcal{V}$ and $j = 1, 2, 3$. In this case, $e_{q,i}(t) \to 0$, which ends the proof.

## V Numerical Simulation

We consider the vehicular platooning with one leader and four followers, where $d_i = 4.46^{\,m} + i$, $h_1 = 2^m$, $h_2 = 1^s$, $\tau_i = 0.7 - 0.1i^s$, $\gamma = 0.1$, $r_i(t) = 0.01\sin(it) + 0.05^{\,s}$, $t \geq 0$, $i = 1, 2, \ldots, 4$, $u_0(t) = 0$, $q_0(0) = 37.84^m$, $q_1(0) = 30.38^m$, $q_2(0) = 21.92^m$, $q_3(0) = 12.46^m$, $q_4(0) = 2^m$, $v_i(0) = 1^{\frac{m}{s}}$, $a_i(0) = 1^{\frac{m^2}{s}}$. We also choose the parameters of observers as $\Upsilon_{i,1} = \Upsilon_{i,4} = \Upsilon_{i,7} = 3$, $\Upsilon_{i,2} = \Upsilon_{i,5} = \Upsilon_{i,8} = 6$, $\Upsilon_{i,3} = \Upsilon_{i,6} = \Upsilon_{i,9} = 4$, $\rho = 1.6$. Based on Lemma 2, the controller parameters (36) are $k_3 = 5$, $k_2 = 3$, $k_1 = 2$, $\vartheta = 0.7$, where $\eta = 1.2$. Two scenarios are considered. In the first one, for the directed topology, we assume the adjacency matrix $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and the pinning matrix $\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Figure 1(a),(c),(d) depicts, respectively, the position, speed, and acceleration trajectories of each vehicle, indicating that vehicle platoon follow trajectories of reference vehicle appropriately. An examination of Fig. 1(b) reveals that the error signal reaches the origin within an exceedingly short transient period, a dynamic behavior that unequivocally attests to the fixed-time consensus




**(a)** First vehicle

**(b)** Second vehicle

**(c)** Third vehicle

**(d)** Fourth vehicle

**Fig. 5.** The position error dynamics with [31] in scenario 1 for directed topology.

mechanism. Figure 1(e) shows the control input in this case. However in theory we have not extensively studied string stability [31], it is investigated in the simulations via the criterion [32]. They represent the string stable behavior. Indeed, a clear monotonic diminution is evident in the actuation signals allocated to each vehicle in the downstream direction. In the second scenario, we set the same parameters for the undirected topology with $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Figure 2 shows satisfactory behavior of the proposed controller. Here, the initial conditions with a larger deviation are set as $q_1(0) = 62.38^m$, $q_2(0) = 45.92^m$, $q_3(0) = 28.46^m$, $q_4(0) = 10^m$, $v_i(0) = 10^{\frac{m}{s}}$, $a_i(0) = 10^{\frac{m^2}{s}}$ without re-tuning other parameters. Comparing Figs. 3 and 4 with Figs. 1 and 2 reveals a negligible difference in convergence time for both directed and undirected topologies, respectively. Notice that the convergence time proves to be irrespective of chosen initial conditions. Then, a comparative evaluation against the approach of [31], which is a PID-type sliding mode strategy with $K_i = 2.8, K_p = 1, K_d = 0$, is conducted, specifically tailored to the directed-topology. Figure 5 depicts the position error dynamics of the vehicles for directed topology, while the position error converges to zero fast in terms of accuracy and settling time.

## VI Conclusions

This study addresses the fixed-time consensus challenge in heterogeneous vehicular platooning systems, explicitly accounting for time-varying delays and parametric uncertainties. As a foundational step, a fixed-time observer is formulated to enable accurate reconstruction of complex unmodeled dynamics. Subsequently, leveraging the observer's estimates, a sliding-mode control law is synthesized to guarantee fixed-time convergence. The proposed observer-based fixed-time control framework is particularly well-suited for cooperative autonomous driving, intelligent transportation systems, and connected vehicle platoons operating under realistic communication delays and uncertainties. Numerical simulations are provided to demonstrate the feasibility of the proposed control scheme.

## References


[1] Hanif, O., Gruber, P., Sorniotti, A., Montanaro, U., A review of control solutions for vehicle platooning via network synchronisation methods, *Automation (2673-4052)*, vol. 7, no. 1, p. 35, 2026.

[2] Li, S.E., Zheng, Y., Li, K., Wang, J., An overview of vehicular platoon control under the four-component framework, In: *IEEE Intelligent Vehicles Symposium (IV)*, IEEE, pp. 286–291, 2015.

[3] Li, S.E., Qin, X., Zheng, Y., Wang, J., Li, K., Zhang, H., Distributed platoon control under topologies with complex eigenvalues: Stability analysis and controller synthesis, *IEEE Transactions on Control Systems Technology*, vol. 27, no. 1, pp. 206–220, 2017.

[4] Han, J., Zhang, J., He, C., Lv, C., Hou, X., Ji, Y., Distributed finite-time safety consensus control of vehicle platoon with senor and actuator failures, *IEEE Transactions on Vehicular Technology*, vol. 72, no. 1, pp. 162–175, 2022.

[5] Elahi, A., Alfi, A., Modares, H., Distributed consensus control of vehicular platooning under delay, packet dropout and noise: Relative state and relative input-output control strategies, *IEEE Transactions on Intelligent Transportation Systems*, vol. 31, no. 11, pp. 20123–20133, 2022.

[6] Zhu, Y., Zhu, F., Distributed adaptive longitudinal control for uncertain third-order vehicle platoon in a networked environment, *IEEE Transactions on Vehicular Technology*, vol. 67, no. 10, pp. 9183–9197, 2018.

[7] Elahi, A., Alfi, A., Modares, H., $H_\infty$ consensus of homogeneous vehicular platooning systems with packet dropout and communication delay, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 6, pp. 3680–3691, 2021.

[8] Li, S.E., *et al.*, Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities, *IEEE Intelligent Transportation Systems Magazine*, vol. 9, no. 3, pp. 46–58, 2017.

[9] Xiao, S., Ge, X., Han, Q. L., Zhang, Y., Dynamic event-triggered platooning control of automated vehicles under random communication topologies and various spacing policies, *IEEE Transactions on Cybernetics*, vol. 52, no. 11, pp. 11477–11490, 2021.

[10] Halder, K., *et al.*, Distributed $H_\infty$ controller design and robustness analysis for vehicle platooning under random packet drop, *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 5, pp. 4373–4386, 2020.

[11] Shahri, E.S.A., Alfi, A., Machado, J.T., Stabilization of fractional-order systems subject to saturation element using fractional dynamic output feedback sliding mode control, *Journal of Computational and Nonlinear Dynamics*, vol. 12, no. 3, p. 031014, 2017.

[12] Esmaeili, N., Alfi, A., Khosravi, H., Balancing and trajectory tracking of two-wheeled mobile robot using backstepping sliding mode control: design and experiments, *Journal of Intelligent & Robotic Systems*, vol. 87, no. 3, pp. 601-613, 2017.

[13] Pahnehkolaei, S.M.A., Alfi, A., Machado, J.T., Fuzzy logic embedding of fractional order sliding mode and state feedback controllers for synchronization of uncertain fractional chaotic systems, *Computational & Applied Mathematics*, vol. 39, no. 3, p. 182, 2020.

[14] Alfi, A., Farrokhi, M., Hybrid state-feedback sliding-mode controller using fuzzy logic for four-wheel-steering vehicles, *Vehicle System Dynamics*, vol. 47, no. 3, pp. 265-284, 2009.

[15] Wu, Y., Li, S.E., Cortés, J., Poolla, K., Distributed sliding mode control for nonlinear heterogeneous platoon systems with positive definite topologies, *IEEE Transactions on Control Systems Technology*, vol. 28, no. 4, pp. 1272–1283, 2019.

[16] Y. Zhu, *et al.*, Joint sliding-mode controller and observer for vehicle platoon subject to disturbance and acceleration failure of neighboring vehicles, *IEEE Transactions on Intelligent Vehicles*, vol. 8, no. 3, pp. 2345–2357, 2022.

[17] Wang, J., Luo, X., Yan, J., Guan, X., Distributed integrated sliding mode control for vehicle platoons based on disturbance observer and multi power reaching law, *IEEE Transactions on intelligent transportation systems*, vol. 23, no. 4, pp. 3366–3376, 2020.

[18] Wang, J., Luo, X., Zhang, Y., Guan, X., Distributed integrated sliding mode control via neural network and disturbance observer for heterogeneous vehicle systems with uncertainties, *Transactions of the Institute of Measurement and Control*, vol. 45, no. 11, pp. 2204–2215, 2023.
[19] Elahi, A., Alfi, A., Finite-time stabilisation of discrete networked cascade control systems under transmission delay and packet dropout via static output feedback control, *International Journal of Systems Science*, vol. 51, no. 1, pp. 87-101, 2020.
[20] Elahi, A., Alfi, A., Puig, V., Stochastic $H_\infty$ fixed-time controller design for discrete-time network-based control systems with time-varying control input delay subject to data loss and disturbances, *International Journal of Systems Science*, vol. 57, no. 8, pp. 2092-2114, 2026.
[21] Zhang, Y., Chadli, M., Xiang, Z., Prescribed-time formation control for a class of multi-agent systems via fuzzy reinforcement learning, *IEEE Transactions on Fuzzy Systems*, vol. 31, no. 12, pp. 4195–4204, 2023.
[22] Yu, Y., Song, Z., Meng, H., Practical prescribed-time sliding-mode output feedback tracking control for heterogeneous vehicle platoon, *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 239, no. 6, pp. 1066–1083, 2025.
[23] Liu, J., Dong, J., Hierarchical prescribed-time platoon control for heterogeneous vehicles with actuator faults and system uncertainties, *IEEE Transactions on Intelligent Transportation Systems*, vol. 26, no. 5, pp. 6965–6976, 2025.
[24] Coppola, A., *et al.*, Distributed fixed-time leader-tracking control for heterogeneous uncertain autonomous connected vehicles platoons, In: *29th Mediterranean Conference on Control and Automation*, pp. 554–559, 2021.
[25] Elahi, A., Alfi, A., Chadli, M., Fixed-time consensus control for uncertain heterogeneous multi-agent systems with high-order dynamics and time-varying delay under generic topologies, *Mathematics and Computers in Simulation*, vol. 225, pp.111-128, 2024.
[26] Karimi, H.R., Guest editorial: Recent advances in sliding mode control under network environment, pp. 247–248, 2022.
[27] Zhang, X.M., Han, Q.L., Yu, X., Survey on recent advances in networked control systems, *IEEE Transactions on Industrial Informatics*, vol. 12, no. 5, pp. 1740–1752, 2015.
[28] Li, H., Cai, Y., On SFTSM control with fixed-time convergence, *IET Control Theory & Applications*, vol. 11, no. 6, pp. 766–773, 2017.
[29] Zuo, Z., Tian, B., Defoort, M., Ding, Z., Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics, *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 563–570, 2017.
[30] Basin, M., Bharath Panathula, C. Shtessel, Y., Multivariable continuous fixed-time second-order sliding mode control: Design and convergence time estimation, *IET Control Theory & Applications*, vol. 11, no. 8, pp. 1104–1111, 2017.
[31] Guo, G., Li, P., Hao, L.Y., A new quadratic spacing policy and adaptive fault-tolerant platooning with actuator saturation, *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 2, pp. 1200–1212, 2020.
[32] Acciani, F., Frasca, P., Heijenk, G., Stoorvogel, A.A., Stochastic string stability of vehicle platoons via cooperative adaptive cruise control with lossy communication, *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 8, pp. 10912–10922, 2021.