Distributed Control For Vehicle Platoon Systems Subject to Heterogeneous Time-Varying Delays

Arezou Elahi[1*]

Abstract: **This study investigates a delay-independent consensus control strategy for vehicular platooning systems subject to time-varying network delays. First, we design a sliding manifold that ensures the convergence of distance tracking errors to zero within a prescribed time interval once sliding occurs. Next, by applying Lyapunov theory, we formulate a distributed control law that steers the system states onto the manifold within a fixed settling time. The theoretical developments are ultimately validated through simulation experiments.**

Keywords: **Distributed control, Vehicular platooning system, Time delay, Uncertainty**

I Introduction

The primary goal of platoon control systems is to ensure that vehicles travel at a common speed while preserving a safe inter-vehicle distance [1]. Such a configuration not only optimizes traffic flow and enhances road safety but also helps reduce overall exhaust emissions [2]. Recent studies have explored various facets of vehicular platooning [3–6]. Within this landscape, communication topology emerges as a pivotal factor in platoon control design [7]. In contrast to adaptive cruise control, cooperative vehicular control systems necessitate both onboard sensing and vehicle-to-vehicle communication to ensure comprehensive safety across the system, thereby enabling superior performance and robust platoon coordination.

Communication topologies for vehicular networks are typically categorized as either directed or undirected [8]. The main feature of our proposed scheme is its topology nature—it operates effectively regardless of whether the graph is directed or undirected, and this versatility serves as the primary impetus for this study. Nevertheless, practical deployments introduce substantial difficulties, including dynamic topology switching, time-varying transmission lags, nonlinear vehicle behavior, and stochastic packet losses. In this context, vehicular platoons can be modeled as multi-agent systems, for which network-based distributed control strategies offer a viable solution. This perspective has given rise to several advanced approaches, such as those in [9, 10].

Sliding mode control (SMC) is extensively utilized in controller synthesis [11, 12] due to its proven efficacy against nonlinearities, uncertainties, topological variations, and disturbances, ensuring precise tracking and fast convergence [13]. Relevant prior works include distributed SMC designs for platoons with various general topologies, to name just a few [14–17].

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The concepts of finite-time and fixed-time stability have been extensively studied for numerous classes of dynamical systems. In these paradigms, the settling time serves as a key performance indicator, measuring how swiftly the controlled response converges [18, 19]. In practice, faster convergence is generally advantageous for improved dynamic behavior. A fundamental drawback of finite-time stabilization, however, lies in its dependence on the initial states when estimating the convergence bound [20–22]. Since these initial conditions are seldom available in multi-agent platforms, this dependency severely limits practical applicability. Fixed-time control overcomes this limitation by decoupling the settling time from the initial conditions [23, 24]. Regrettably, however, nearly all published works in this area assume perfect network conditions, effectively disregarding transmission delays and other communication imperfections [25, 26]. The present study bridges this gap by formulating a fixed-time controller for vehicle platoons subject to time-varying communication delays. To the best of our knowledge, no existing work has addressed the fixed-time consensus control problem for heterogeneous vehicular platoons under simultaneous time-varying communication delays and parametric uncertainties using an observer-based sliding-mode framework.

Motivated by the aforementioned discussions, this work develops a consensus control scheme for heterogeneous platoons under general topologies by designing an observer-based predefined-time sliding-mode controller. The proposed design rigorously addresses time-varying communication delays and system uncertainties, making it directly applicable to realistic scenarios where such network-induced imperfections are unavoidable.

The remainder of this paper is organized as follows. Section II introduces the problem formulation and preliminaries. Section III presents the proposed observer, the controller, and the associated stability analysis. Numerical simulations are provided in Section IV, followed by concluding remarks in Section V.

We employ the following mathematical notations: $\mathbb{R}^n$ and $\mathbb{R}^{N\times N}$ denote, respectively, the sets of real n-component vectors and real $N \times N$ matrices, $\lambda_{\max}$ and

$\lambda_{\min}$ are the maximum and minimum eigenvalues of a given matrix, I_n indicates the identity matrix of order n, and for $\sigma \geq 0$, $x \in \mathbb{R}^n$, the function $\lceil y \rfloor^{\sigma} = |x|^{\sigma}\text{sign}(y)$ is used. Finally, $\|X\|$ is adopted throughout to represent the Euclidean norm of a vector or matrix X.

II Problem Formulation

First, we state the preliminaries that will be used in the manuscript.

A Graph Theory

Suppose that the symbol $G = (\mathcal{V}, \mathcal{E})$ shows the communication topology among N followers, where $\mathcal{V} = \{1, 2, ..., N\}$ denotes the set of followers and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represents the set of edges. N_i indicates the set of neighbors of follower i. If there is a link from follower j to follower i, then $(j, i) \in \mathcal{E}$. In addition, $A = [a_{ij}]$ denotes the adjacency matrix in which if $(i, j) \in \mathcal{E}$, then $a_{i,j} > 0$, otherwise $a_{ij} = 0$. The Laplacian matrix is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where the elements satisfy $l_{ii} = \sum_{j\in N} a_{ij}$, and $l_{ij} = -a_{ij}$, where $i \neq j$. Define $\mathcal{M} = diag\,(g_i)$ as the diagonal pinning matrix, where $g_i = 1$ if we have a directed edge from the leader to the follower, and $g_i = 0$ otherwise. Notice that there is at least one edge connecting leader and followers.

Lemma 1 [27]: Let us consider the system with dynamical equation

$$\dot{\xi}(t) = -\lambda_1 \lceil \xi(t) \rfloor^{\rho_1} - \lambda_2 \lceil \xi(t) \rfloor^{\rho_2} \tag{1}$$

with $\lceil \xi(t) \rfloor^{\rho_1} = \text{sign}\,(\xi(t))|\xi(t)|^{\rho_1}$, λ_1, $\lambda_2 > 0$, $1 > \rho_2 > 0$, $\rho_1 > 1$. Thus, for the system (1), fixed- stability is guaranteed with the maximum admissible value of setting time $T = \frac{1}{\lambda_1(\rho_1-1)} + \frac{1}{\lambda_2(1-\rho_2)}$.

Lemma 2 [28]: Let us consider the following integrator system with the initial conditions $\xi(0) = \xi_0$

$$\begin{aligned}\dot{\xi}_1(t) &= \xi_2(t)\\ \dot{\xi}_2(t) &= \xi_3(t)\\ \dot{\xi}_3(t) &= u(t)\end{aligned} \tag{2}$$

where $\xi_i(t) \in \mathbb{R}^n$, $i = 1, 2, 3$, and $u(t)$ represent the system's states and the control signal, respectively. Assume the polynomial $s^3 + k_3 s^2 + k_2 s + k_1$ is Hurwitz, in which s denotes the Laplace operator and $k_i > 0$, $i = 1, 2, 3$ are the positive constants. There exists a scalar $\check{\gamma} \in (\frac{1}{2}, 1)$ so that, for $\vartheta \in (\check{\gamma}, 1)$, the system (2) in a fixed-time is stabilized at the origin via the following control law

$$u(t) = -\sum_{i=1}^{3} k_i \left(\lceil \xi_i(t) \rfloor^{\alpha_i} + \lceil \xi_i(t) \rfloor + \lceil \xi_i(t) \rfloor^{\tilde{\alpha}_i} \right) \tag{3}$$

in which the parameters α_i and $\tilde{\alpha}_i$ satisfying $\alpha_{3-j} = \frac{\vartheta}{(j+1)-j\vartheta}$, $\tilde{\alpha}_{3-j} = \frac{2-\vartheta}{j\vartheta-(j-1)}$, $j = 0, 1, 2$.

Lemma 3 [28]: For the positive parameters η_i, $i = 1, ..., N$, $1 \geq \alpha_1 > 0$, $\alpha_2 > 1$, the following relationships hold

$$\begin{aligned} N^{1-\alpha_2}\left(\sum_{i=1}^{N} \eta_i\right)^{\alpha_2} &\leq \sum_{i=1}^{N} \eta_i^{\alpha_2}\\ \left(\sum_{i=1}^{N} \eta_i\right)^{\alpha_1} &\leq \sum_{i=1}^{N} \eta_i^{\alpha_1} \end{aligned} \tag{4}$$

B Problem statement

Similar to most of the existing literature, we adopt the heterogeneous third-order model for i-th vehicle as [?]

$$\begin{aligned}\dot{q}_i(t) &= v_i(t)\\ \dot{v}_i(t) &= a_i(t)\\ \dot{a}_i(t) &= -\frac{1}{\tau_i}a_i(t) + \frac{1}{\tau_i}u_i(t) + f_i(t, q_i, v_i, a_i)\end{aligned} \tag{5}$$

where $i \in \{0, 1, \ldots, N\}$, a_i, v_i, $q_i \in \mathbb{R}^n$ represent the acceleration, velocity, and position, respectively, $u_i(t)$, τ_i indicate the control input and the time constant, respectively, and the uniformly bounded function f_i satisfying $|f_i| \leq \gamma$ denotes the uncertainty term, where γ is a certain positive constant. The rest of this article, $f_i(t)$ will adopted instead of $f_i(t, q_i, v_i, a_i)$.

Let us consider the spacing error

$$e_{q,i}(t) = \Big(-q_i(t)+q_{i-1}(t-r_i(t))-d_i\Big)-\Big(h_1+h_2 v_i(t)\Big) \tag{6}$$

in which h_1, h_2, d_i describe the standstill distance, desired time gap, and vehicle length, respectively. The transmission delays $r_i\,(t)$ between the vehicles $i-1$ and i is time-varying and unknown bounded.

The central goal of the fixed-time consensus control problem is the derivation of control policies that enforce the consensus error states to vanish within a uniformly bounded time, independent of the system's initialization. Upon differentiating the spacing errors presented in (6), one can derive the error dynamics outlined below.

$$\begin{aligned}\dot{e}_{q,i}(t) &= \varsigma_{i,1}(t) + v_{i-1}(t - r_i(t)) + v_{i-1}(t)\\ &\quad - v_i(t) - h_2\, a_i(t)\\ \dot{e}_{v,i}(t) &= \varsigma_{i,2}(t) + a_{i-1}(t - r_i(t)) + a_{i-1}(t)\\ &\quad - a_i(t) - h_2\, \dot{a}_i(t)\\ \dot{e}_{a,i}(t) &= \varsigma_{i,3}(t) + \dot{a}_{i-1}(t) - \dot{a}_i(t) - h_2\, \ddot{a}_i(t)\end{aligned} \tag{7}$$

where

$$\begin{aligned}e_{v,i}(t) &= v_{i-1}(t - r_i(t)) + v_{i-1}(t) - v_i(t) - h_2\, a_i(t)\\ e_{a,i}(t) &= a_{i-1}(t - r_i(t)) + a_{i-1}(t) - a_i(t) - h_2\, \dot{a}_i(t)\\ \varsigma_{i,1}(t) &= -\dot{r}_i(t)v_{i-1}(t - r_i(t)) - v_{i-1}(t)\\ \varsigma_{i,2}(t) &= -\dot{r}_i(t)a_{i-1}(t - r_i(t))\\ \varsigma_{i,3}(t) &= -\dot{r}_i(t)\, \dot{a}_{i-1}(t - r_i(t)) + \dot{a}_{i-1}(t - r_i(t))\end{aligned} \tag{8}$$

Through straightforward simplifications, the following expression is obtained

$$\begin{aligned}\dot{e}_{q,i}(t) &= e_{v,i}(t) + \varsigma_{i,1}(t)\\ \dot{e}_{v,i}(t) &= e_{a,i}(t) + \varsigma_{i,2}(t)\\ \dot{e}_{a,i}(t) &= \Omega_i(t) + \breve{u}_i(t) + \hat{\varsigma}_{i,3}(t)\end{aligned} \tag{9}$$

where

$$\hat{\varsigma}_{i,3}(t) = \varsigma_{i,3}(t) + f_{i-1}(t) - f_i(t) + \frac{h_2}{\tau_i} f_i(t) - h_2 \dot{f}_i(t)$$
$$\breve{u}_i(t) = \Big(\frac{1}{\tau_{i-1}} u_{i-1}(t) - \frac{1}{\tau_i} u_i(t) + \frac{h_2}{\tau_i^2} u_i(t) - \frac{h_2}{\tau_i} \dot{u}_i(t)\Big)$$
$$\Omega_i(t) = \Big(- \frac{1}{\tau_{i-1}} a_{i-1}(t) + \frac{1}{\tau_i} a_i(t) - \frac{h_2}{\tau_i^2} a_i(t)\Big) \tag{10}$$

III Stability Analysis

Note from (7)–(10) that $\varsigma_{i,1}$, $\varsigma_{i,2}$, $\hat{\varsigma}_{i,3}$ denote complex unknowns, including uncertainties and immeasurable $\dot{r}\,(t)$, that inefficient mitigation of these disturbances risks degrading the consensus capability. For this purpose, a fixed-time observer is presented in the sequel to facilitate exact observation.

Theorem 1: Applying the error dynamics (9), suppose that the complex unknowns $\varsigma_{i,1}(t)$, $\varsigma_{i,2}(t)$, $\hat{\varsigma}_{i,3}(t)$ are respectively bounded by $\|\dot{\varsigma}_{i,1}(t)\|_< l_{i,1}$, $\|\dot{\varsigma}_{i,2}(t)\|_< l_{i,2}$, $\|\dot{\hat{\varsigma}}_{i,3}(t)\|_< l_{i,3}$, where $l_{i,j}$, $j = 1, 2, 3$ are positive constants. Under these conditions, the fixed-time observers are designed as

$$\dot{x}_{i,0}(t) = -\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|^{\rho-1} + x_{i,1}(t) + e_{i,v}(t), \qquad x_{i,0}(0) = 2e_{q,i}(0) \tag{11}$$

$$\dot{x}_{i,1}(t) = -\Upsilon_{i,3} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|}, \qquad x_{i,1}(0) = \varsigma_{i,1}(0) \tag{12}$$

$$\dot{x}_{i,2}(t) = -\Upsilon_{i,4} \frac{n_{i,2}(t)}{\|n_{i,2}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,5}\, n_{i,2}(t) \|n_{i,2}(t)\|^{\rho-1} + x_{i,3}(t) + e_{i,a}(t), \qquad x_{i,2}(0) = 2e_{v,i}(0) \tag{13}$$

$$\dot{x}_{i,3}(t) = -\Upsilon_{i,6} \frac{n_{i,2}(t)}{\|n_{i,2}(t)\|}, \qquad x_{i,3}(0) = \varsigma_{i,2}(0) \tag{14}$$

$$\dot{x}_{i,4}(t) = -\Upsilon_{i,7} \frac{n_{i,4}(t)}{\|n_{i,4}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,8}\, n_{i,4}(t) \|n_{i,4}(t)\|^{\rho-1} + x_{i,5}(t) + \Omega_i(t) + \breve{u}_i(t), \qquad x_{i,4}(0) = 2e_{a,i}(0) \tag{15}$$

$$\dot{x}_{i,5}(t) = -\Upsilon_{i,9} \frac{n_{i,4}(t)}{\|n_{i,4}(t)\|}, \qquad x_{i,5}(0) = \hat{\varsigma}_{i,3}(0) \tag{16}$$

where $\rho > 1$, $x_{i,j}$, $j = 1, ..., 5$ are the state variable of observers, and the auxiliary variables are established as $n_{i,0}(t) = x_{i,0}(t) - e_{q,i}(t), n_{i,1} = x_{i,1}(t) - \varsigma_{i,1}(t)$, $n_{i,2}(t) = x_{i,2}(t) - e_{v,i}(t)$, $n_{i,3}(t) = x_{i,3}(t) - \varsigma_{i,2}(t)$, $n_{i,4}(t) = x_{i,4}(t) - e_{a,i}(t), n_{i,5}(t) = x_{i,5}(t) - \hat{\varsigma}_{i,3}(t)$, and the parameters are satisfied, where

$$\Upsilon_{i,7} > \sqrt{2\Upsilon_{i,9}},\ \Upsilon_{i,8} > 0,\ \Upsilon_{i,9} > 4\, l_{i,3} \tag{17}$$

$$\Upsilon_{i,4} > \sqrt{2\Upsilon_{i,6}},\ \Upsilon_{i,5} > 0,\ \Upsilon_{i,6} > 4 l_{i,2} \tag{18}$$

$$\Upsilon_{i,1} > \sqrt{2\Upsilon_{i,3}},\ \Upsilon_{i,2} > 0,\ \Upsilon_{i,3} > 4 l_{i,1} \tag{19}$$

Then, the mismatched disturbances $\varsigma_{i,1}(t)$, $\varsigma_{i,2}(t)$, and unknown term $\hat{\varsigma}_{i,3}(t)$ is exactly observed within a fixed-time independent of the initial states. That is, there exists a fixed-time T_{fj}, $j = 1, 2, 3$, so that, $x_{i,1} = \varsigma_{i,1}$, $x_{i,3} = \varsigma_{i,2}$, and $x_{i,5} = \hat{\varsigma}_{i,3}$, for $t \geq T_0$, where $T_0 = \max(T_{f1}, T_{f2}, T_{f3})$.

Proof: We first present the criteria that ensure finite-time convergence. Building on this, we prove that the system converges within a fixed time. Applying (6), (11), and (12), we have

$$\begin{aligned}\dot{n}_{i,0}(t) &= -\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|\|^{\rho-1} \\ &\quad + x_{i,1}(t) + e_{i,v}(t) - e_{i,v}(t) - \varsigma_{i,1}(t) \\ &= -\Upsilon_{i,1} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}} - \Upsilon_{i,2} n_{i,0}(t) \|n_{i,0}(t)\|^{\rho-1} \\ &\quad + n_{i,1}(t), \quad n_{i,0}(0) = e_{i,q}(0)\end{aligned} \tag{20}$$

$$\dot{n}_{i,1}(t) = -\Upsilon_{i,3} \frac{n_{i,0}(t)}{\|n_{i,0}(t)\|} - \dot{\varsigma}_{i,1}(t), \quad n_{i,1}(0) = 0 \tag{21}$$

In the first step, we establish the following Lyapunov function for (20) and (21)

$$V(t) = 2\Upsilon_{i,3} \left\|n_{i,0}(t)\right\| + \frac{1}{2} \left\|n_{i,1}(t)\right\|^2 + \frac{1}{2} \left\|\Upsilon_{i,1} \frac{n_{i,0}(t)}{\left\|n_{i,0}(t)\right\|^{\frac{1}{2}}} - n_{i,1}(t)\right\|^2 \tag{22}$$

Through straightforward algebraic rearrangement, the derivative of (22) with respect to time is given by

$$\dot{V}(t) \leq -\frac{z^T \psi z}{\|n_{i,0}(t)\|^{\frac{1}{2}}} \leq -\lambda_{\min}(\psi) \frac{\|z\|^2}{\|n_{i,0}(t)\|^{\frac{1}{2}}} \tag{23}$$

where $z = col(\|n_{i,0}(t)\|^{\frac{1}{2}}, \|n_{i,1}(t)\|)$ and the matrix ψ is positive definite, if

$$\Upsilon_{i,1}^2 > 2l_{i,1},\ \ \Upsilon_{i,3} \geq 3l_{i,1} + \frac{2l_{i,1}^2}{\Upsilon_{i,1}^2} \tag{24}$$

It is worth emphasizing that condition (24) is not only sufficient for guaranteeing the convergence of the observer error dynamics, but also guarantees the positive definiteness of matrix ψ. Specifically, by selecting the observer gains such that $\Upsilon_{i,1}$ and $\Upsilon_{i,3}$ dominate the corresponding disturbance upper bounds, every principal minor of ψ becomes strictly positive, yielding $\lambda_{min}(\psi) > 0$. Then, defining $Z = col\left(\frac{n_{i,0}(t)}{\|n_{i,0}(t)\|^{\frac{1}{2}}}, \|n_{i,1}(t)\|\right)$, due to since $\|Z\| = \|z\|$, we have

$$\dot{V}(t) \leq -\lambda_{\min}(\psi) \frac{\|Z\|^2}{\|n_{i,0}(t)\|^2} \tag{25}$$

Considering a symmetric positive-definite matrix $\mathcal{P}$

$$\mathcal{P} = \begin{bmatrix} \left(2\Upsilon_{i,3} + \frac{\Upsilon_{i,1}^2}{2}\right) I_n & -\frac{\Upsilon_{i,1}}{2} I_n \\ -\frac{\Upsilon_{i,1}}{2} I_n & I_n \end{bmatrix} \tag{26}$$

and constructing the Lyapunov function as $V = Z^T \mathcal{P} Z$, we have $\sqrt{\lambda_{\min}(\mathcal{P})}\|Z\| \leq V^{\frac{1}{2}} \leq \sqrt{\lambda_{\max}(\mathcal{P})}\|Z\|$ and then

$$\dot{V} \leq -\hat{\rho} V^{\frac{1}{2}} \tag{27}$$

in which $\hat{\rho} = \frac{\lambda_{\min}(\psi)\sqrt{\lambda_{\min}(\mathcal{P})}}{\lambda_{\max}(\mathcal{P})}$. From (27), we achieve the convergence of the systems' states (20) and (21) to the origin. Notice that due to $\Upsilon_{i,1}^2 > 2l_{i,1}$, we have $\frac{1}{\Upsilon_{i,1}^2} < \frac{1}{2l_{i,1}}$ the second condition in (24) holds if $\Upsilon_{i,3} > 4l_{i,1}$. Given that $\Upsilon_{i,3} > l_{i,1}$, the first condition is given by $\Upsilon_{i,1}^2 > 2\Upsilon_{i,3}$. Therefore, the finite-time convergence conditions (24) is expressed in the form $\Upsilon_{i,3} > 4l_{i,1}$ and $\Upsilon_{i,1}^2 > 2\Upsilon_{i,3}$.

In the second step, invoking the findings of [29], we deduce that the signals $n_{i,0}(t)$ and $n_{i,1}(t)$ in (20) and (21) are uniformly driven to zero over a fixed time horizon. That is, there exists a fixed-time T_{f1} irrespective of initializations such that $n_{i,0}(t) = n_{i,1}(t) = 0,\ t \geq T_{f1}$. Because of the similarity, the proof of $n_{i,3}(t) = 0,\ t \geq T_{f2}$ and $n_{i,5}(t) = 0,\ t \geq T_{f3}$, are omitted herein. It follows that disturbances $\varsigma_{i,1}(t),\ \varsigma_{i,2}(t),\ \hat{\varsigma}_{i,3}(t)$ can be exactly observed with in $T_0 = \max(T_{f1},\ T_{f2},\ T_{f3})$. This completes the proof.

Remark 1: The observer gains $\Upsilon_{i,1}$ and $\Upsilon_{i,3}$ should be selected to be substantially larger than the estimated upper bounds of the disturbances. Larger gains typically accelerate observer convergence; however, excessively conservative selections may result in increased control effort. Thus, a balanced gain selection provides a practical compromise between convergence performance and implementation robustness. The constants $l_{i,1}$, $l_{i,2}$, and $l_{i,3}$ similarly denote conservative upper bounds on the lumped uncertainties affecting the observer dynamics. In practice, these bounds can be estimated using available system information, experimental identification, engineering specifications, or worst-case operating conditions. It is important to emphasize that the proposed observer does not require precise knowledge of the uncertainty magnitudes. Instead, sufficiently conservative upper bounds are adequate to ensure the theoretical convergence properties. Although overestimating these bounds may lead to larger observer gains and, consequently, heightened control effort, the fixed-time convergence of the observer remains guaranteed.

Next, a fixed-time control scheme is constructed using the fixed-time observer established above. Defining $p_{i,1} = e_{q,i}(t)$, $p_{i,2}(t) = x_{i,1} + e_{v,i}(t)$, $p_{i,3}(t) = \dot{x}_{i,1}(t) + x_{i,3} + e_{a,i}(t)$, the error dynamics (9) is rewritten as

$$\dot{p}_{i,1}(t) = p_{i,2}(t) \tag{28}$$

$$\dot{p}_{i,2}(t) = p_{i,3}(t) \tag{29}$$

$$\dot{p}_{i,3}(t) = \Omega_i + \breve{u}_i(t) + \ddot{x}_{i,1}(t) + \dot{x}_{i,3}(t) + x_{i,5}(t) \tag{30}$$

For developing fixed-time consensus control of vehicles, the following new variables are defined.

$$e_{p_{i,1}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,1}(t) - p_{j,1}(t)) + g_i p_{i,1}(t) \tag{31}$$

$$e_{p_{i,2}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,2}(t) - p_{j,2}(t)) + g_i p_{i,2}(t) \tag{32}$$

$$e_{p_{i,3}}(t) = \sum_{j=1}^{N} a_{ij}(p_{i,3}(t) - p_{j,3}(t)) + g_i p_{i,3}(t) \tag{33}$$

For the i-th agent, the sliding manifold is proposed as

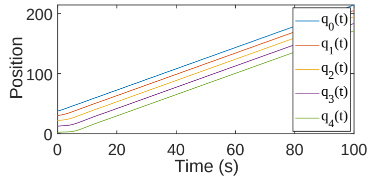
$$\begin{aligned} \sigma_i(t) = p_{i,3}(t) &+ k_1 \int_0^t \left(\lceil e_{p_{i,1}}(\theta) \rfloor^{\alpha_1} + e_{p_{i,1}}(\theta) + \lceil e_{p_{i,1}}(\theta) \rfloor^{\tilde{\alpha}_1} \right) d\theta \\ &+ k_2 \int_0^t \left(\lceil e_{p_{i,2}}(\theta) \rfloor^{\alpha_2} + e_{p_{i,2}}(\theta) + \lceil e_{p_{i,2}}(\theta) \rfloor^{\tilde{\alpha}_2} \right) d\theta \\ &+ k_3 \int_0^t \left(\lceil e_{p_{i,3}}(\theta) \rfloor^{\alpha_3} + e_{p_{i,3}}(\theta) + \lceil e_{p_{i,3}}(\theta) \rfloor^{\tilde{\alpha}_3} \right) d\theta \end{aligned} \tag{34}$$

where the scalars $k_i,\ \alpha_i,\ \tilde{\alpha}_i,\ i = 1, 2, 3$, are opted based on Lemma 2. Differentiating (34) yields

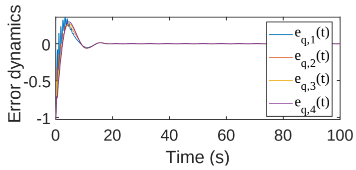
$$\begin{aligned} \dot{\sigma}_i(t) = \dot{p}_{i,3}(t) &+ k_1 \left(\lceil e_{p_{i,1}}(t) \rfloor^{\alpha_1} + e_{p_{i,1}}(t) + \lceil e_{p_{i,1}}(t) \rfloor^{\tilde{\alpha}_1} \right) \\ &+ k_2 \left(\lceil e_{p_{i,2}}(t) \rfloor^{\alpha_2} + e_{p_{i,2}}(t) + \lceil e_{p_{i,2}}(t) \rfloor^{\tilde{\alpha}_2} \right) \\ &+ k_3 \left(\lceil e_{p_{i,3}}(t) \rfloor^{\alpha_3} + e_{p_{i,3}}(t) + \lceil e_{p_{i,3}}(t) \rfloor^{\tilde{\alpha}_3} \right) \end{aligned} \tag{35}$$

Considering the error dynamics (30) and sliding manifold (34) with $\eta > 1$, we can calculate the control signal as

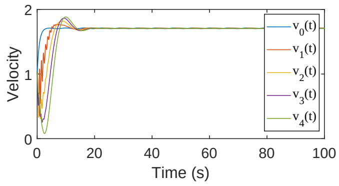
$$\begin{aligned} \breve{u}_i(t) = &- \left(\Omega_i(t) + \ddot{x}_{i,1}(t) + \dot{x}_{i,3}(t) + x_{i,5}(t) \right) \\ &- k_3 \left(\lceil e_{p_{i,3}}(t) \rfloor^{\alpha_3} + e_{p_{i,3}}(t) + \lceil e_{p_{i,3}}(t) \rfloor^{\tilde{\alpha}_3} \right) \\ &- k_2 \left(\lceil e_{p_{i,2}}(t) \rfloor^{\alpha_2} + e_{p_{i,2}}(t) + \lceil e_{p_{i,2}}(t) \rfloor^{\tilde{\alpha}_2} \right) \\ &- k_1 \left(\lceil e_{p_{i,1}}(t) \rfloor^{\alpha_1} + e_{p_{i,1}}(t) + \lceil e_{p_{i,1}}(t) \rfloor^{\tilde{\alpha}_1} \right) \\ &- \left(\lceil \sigma_i(t) \rfloor^{1+\frac{1}{\eta}} + \sigma_i(t) + \lceil \sigma_i(t) \rfloor^{1-\frac{1}{\eta}} \right) \end{aligned} \tag{36}$$



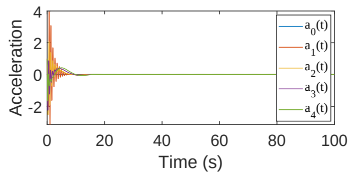
(a) Position



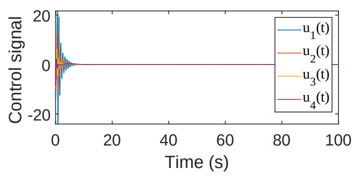
(b) Position error



(c) Veocity

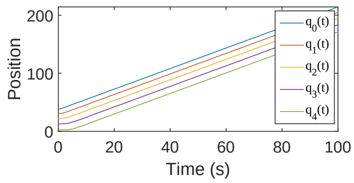


(d) Acceeration

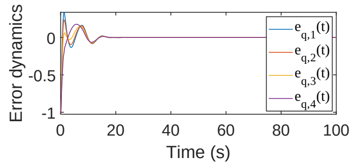


(e) Control signal

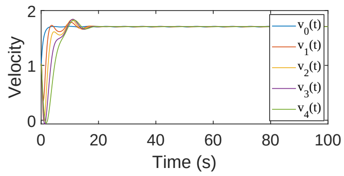
Fig. 1. Trajectories of directed topology in scenario 1.



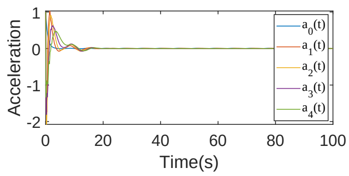
(a) Position



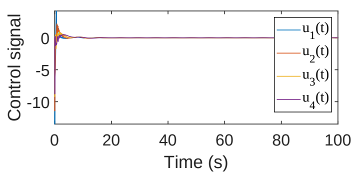
(b) Position error



(c) Veocity



(d) Acceeration



(e) Control signal

Fig. 2. Trajectories of undirected topology in scenario 1.

Remark 1. *Based on incomplete measurements, we introduce an observer* (11)–(16) *to observe the disturbance signals and unknown functions within a fixed-time. From Theorem 1, the mismatched disturbances* $\varsigma_{i,1}$*,* $\varsigma_{i,2}$ *and unknown term* $\hat{\varsigma}_{i,3}$ *is observed within a fixed time if* (19)–(17) *hold. Thus,* $x_{i,1} = \varsigma_{i,1}$*,* $x_{i,3} = \varsigma_{i,2}$*,* $x_{i,5} = \hat{\varsigma}_{i,3}$ *is applied in the control design scheme.*

Here, independent of the graph topology, we assume that $(\mathcal{L} + \mathcal{M})$ is a positive definite matrix.

Theorem 1. *For the error dynamics* (28)-(30) *under Assumption, applying the observer* (11)-(16) *and the control signal* (36)*, the error dynamics converge to zero in a predefined time independent of initializations.*

Proof: The derivative of Lyapunov function

$$V(t) = \sum_{i=1}^{N} 0.5\sigma_i^2(t) \tag{37}$$

with respect to time is given by

$$\dot{V}(t) = \sum_{i=1}^{N} \sigma_i(t)\dot{\sigma}_i(t) \tag{38}$$

From (30), (35), and (36), we get

$$\dot{V}(t) = -\sum_{i=1}^{N} \sigma_i(t) \left(\lceil \sigma_i(t) \rfloor^{1-\frac{1}{\eta}} + \lceil \sigma_i(t) \rfloor^{1+\frac{1}{\eta}} + \sigma_i(t) \right) \tag{39}$$

that satisfy

$$\dot{V}(t) \leq -\sum_{i=1}^{N} \left(|\sigma_i(t)|^{2+\frac{1}{\eta}} + \left|\sigma_i(t)\right|^2 + |\sigma_i(t)|^{2-\frac{1}{\eta}} \right) \tag{40}$$

Applying Lemma 3, it yields

$$\dot{V}(t) \leq -(V^{1-\frac{1}{2\eta}} + N^{\frac{-1}{2\eta}} V^{1+\frac{1}{2\eta}}) \tag{41}$$

According to Lemmas 1 and 2, we ensure that every agent reaches the sliding manifold $\sigma_i(t) = 0$ within $T_r \leq 2\eta\,(1 + N^{\frac{1}{2\eta}})$ and stays there subsequently independent of initializations. In the sliding motion, $\sigma_i(t) = 0$ yields $\dot{\sigma}_i(t) = 0$. Consequently, the reduced resultant dynamics, for $t \geq T_0 + T_r$, is given by

$$
\begin{aligned}
\dot{e}_{p_1}(t) &= e_{p_2}(t)\\
\dot{e}_{p_2}(t) &= e_{p_3}(t)\\
\dot{e}_{p_3}(t) &= -\tilde{\mathcal{L}}K_1\left(\left\lceil e_{p_1}(t)\right\rfloor^{\alpha_1} + e_{p_1}(t) + \left\lceil e_{p_1}(t)\right\rfloor^{\tilde{\alpha}_1}\right)\\
&\quad - \tilde{\mathcal{L}}K_2\left(\left\lceil e_{p_2}(t)\right\rfloor^{\alpha_2} + e_{p_2}(t) + \left\lceil e_{p_2}(t)\right\rfloor^{\tilde{\alpha}_2}\right)\\
&\quad - \tilde{\mathcal{L}}K_3\left(\left\lceil e_{p_3}(t)\right\rfloor^{\alpha_3} + e_{p_3}(t) + \left\lceil e_{p_3}(t)\right\rfloor^{\tilde{\alpha}_3}\right)
\end{aligned} \tag{42}
$$

where $\tilde{\mathcal{L}} = (\mathcal{L} + \mathcal{M})$, and for $j = 1, 2, 3$,

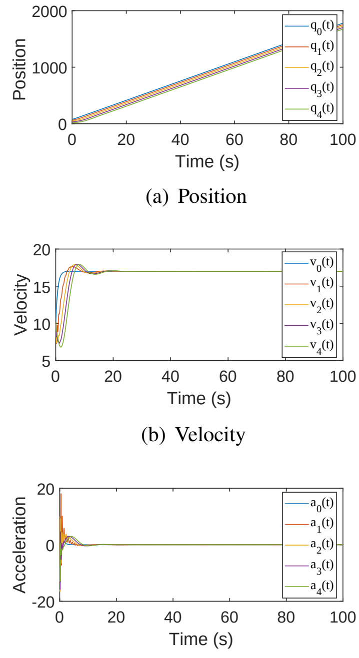
$$
e_{p_j} = \begin{bmatrix} e_{p_{1,j}} \\ e_{p_{2,j}} \\ \vdots \\ e_{p_{N,j}} \end{bmatrix},\ \left\lceil e_{p_j}(t)\right\rfloor^{\sigma} = \begin{bmatrix} \left\lceil e_{p_{1,j}}(t)\right\rfloor^{\sigma} \\ \left\lceil e_{p_{2,j}}(t)\right\rfloor^{\sigma} \\ \vdots \\ \left\lceil e_{p_{N,j}}(t)\right\rfloor^{\sigma} \end{bmatrix},\ \sigma = \alpha_j, \tilde{\alpha}_j
$$

$$
K_j = \begin{bmatrix} k_j & 0 & \cdots & 0 \\ 0 & k_j & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & k_j \end{bmatrix} \in \mathbb{R}^{N\times N} \succ 0 \tag{43}
$$

From Lemma 2, (42) are fixed-time stable at the origin. In other words, irrespective of the initializations and for $t \geq T_r + T_0$, we have $e_{p_j} \rightarrow 0$, $j = 1, 2, 3$, for all agents, where T_0 is given by in Theorem 1. Since $e_{p_j} \rightarrow 0$ indicates $p_{i,j} \rightarrow 0$, $i \in \mathcal{V}$ and $j = 1, 2, 3$. In this case, $e_{q,i}(t) \rightarrow 0$, which ends the proof.

IV Numerical Simulation

We consider the vehicular platooning with one leader and four followers, where $d_i = 4.46^{m} + i$, $h_1 = 2^m$, $h_2 = 1^s$, $\tau_i = 0.7 - 0.1i^s$, $\gamma = 0.1$, $r_i(t) = 0.01\sin(it) + 0.05^{s}$, $t \geq 0$, $i = 1, 2, \ldots, 4$, $u_0(t) = 0$, $q_0(0) = 37.84^m$, $q_1(0) = 30.38^m$, $q_2(0) = 21.92^m$, $q_3(0) = 12.46^m$, $q_4(0) = 2^m$, $v_i(0) = 1^{\frac{m}{s}}$, $a_i(0) = 1^{\frac{m^2}{s}}$. We also choose the parameters of observers as $\Upsilon_{i,1} = \Upsilon_{i,4} = \Upsilon_{i,7} = 3$, $\Upsilon_{i,2} = \Upsilon_{i,5} = \Upsilon_{i,8} = 6$, $\Upsilon_{i,3} = \Upsilon_{i,6} = \Upsilon_{i,9} = 4$, $\rho = 1.6$. Based on Lemma 2, the controller parameters (36) are $k_3 = 5$, $k_2 = 3$, $k_1 = 2$, $\vartheta = 0.7$, where $\eta = 1.2$. Two scenarios are considered. In the first one, for the directed topology, we assume the adjacency matrix $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and the pinning matrix $\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.



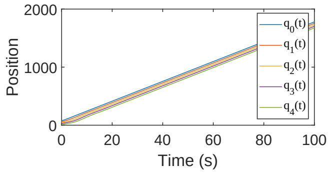
(a) Position

(b) Velocity

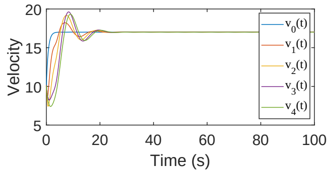
(c) Acceleration

Fig. 3. Trajectories of directed topology in scenario 2.

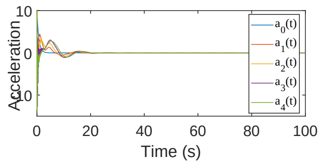
Figure 1(a),(c),(d) depicts, respectively, the position, speed, and acceleration trajectories of each vehicle, indicating that vehicle platoon follow trajectories of reference vehicle appropriately. An examination of Fig. 1(b) reveals that the error signal reaches the origin within an exceedingly short transient period, a dynamic behavior that unequivocally attests to the fixed-time consensus mechanism. Figures 1(e) and 2(e) show the control input in this case. However in theory we have not extensively studied string stability [30], it is investigated in the simulations via the criterion [31]. They represent the string stable behavior. Indeed, a clear monotonic diminution is evident in the actuation signals allocated to each vehicle in the downstream direction. In the second scenario, we set the same parameters for the undirected topology with $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Figure 2 shows satisfactory behavior of the proposed controller. Here, the initial conditions with a larger deviation are set as $q_1(0) = 62.38^m$, $q_2(0) = 45.92^m$, $q_3(0) = 28.46^m$, $q_4(0) = 10^m$, $v_i(0) = 10^{\frac{m}{s}}$, $a_i(0) = 10^{\frac{m^2}{s}}$ without re-tuning other parameters. Comparing Figs. 3 and 4 with Figs. 1 and 2 reveals a negligible difference in convergence time for both directed and undirected topologies, respectively. Notice that the convergence time proves to be irrespective of chosen initial conditions. Then, a comparative evaluation against the approach of [30] , which is a PID-type sliding mode strategy with $K_i = 2.8$, $K_p = 1$, $K_d = 0$, is conducted, specifically tailored to the directed-topology. Figure 5 depicts the position error dynamics of the vehicles for directed topology, while the position error converges to zero fast in terms of accuracy and settling time.



(a) Position

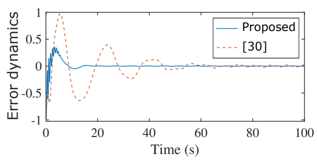


(b) Velocity

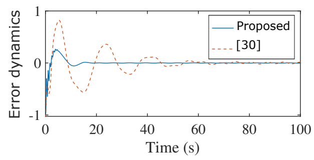


(c) Acceleration

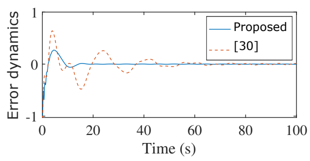
Fig. 4. Trajectories of undirected topology in scenario 2.



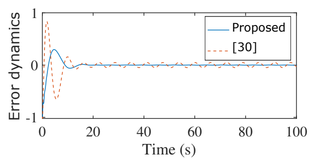
(a) First vehicle



(b) Second vehicle



(c) Third vehicle



(d) Forth vehicle

Fig. 5. The position error dynamics with [30] in scenario 1 for directed topology.

V Conclusions

This study addresses the fixed-time consensus challenge in heterogeneous vehicular platooning systems, explicitly accounting for time-varying delays and parametric uncertainties. As a foundational step, a fixed-time observer is formulated to enable accurate reconstruction of complex unmodeled dynamics. Subsequently, leveraging the observer's estimates, a sliding-mode control law is synthesized to guarantee fixed-time convergence. The proposed observer-based fixed-time control framework is particularly well-suited for cooperative autonomous driving, intelligent transportation systems, and connected vehicle platoons operating under realistic communication delays and uncertainties. Numerical simulations are provided to demonstrate the feasibility of the proposed control scheme.

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